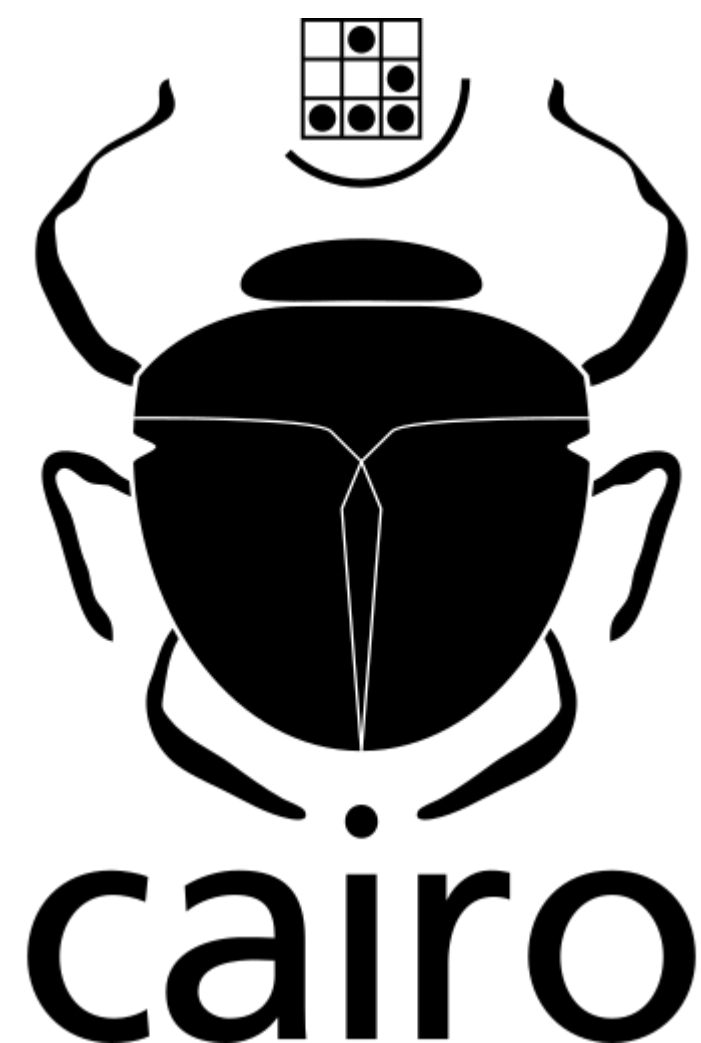


Motivation

The goal of the project is the extension of a pre-existing 2D physics engine implemented by Prof. Carzaniga. That engine, called "flying-balls", only simulated interactions between circles. They bounced off of each other and off the walls represented by the edges of the window they were evolving in. The extension decided was to add the possibility to make arbitrary polygons collide with each other similarly to the balls in the initial project.

Technologies

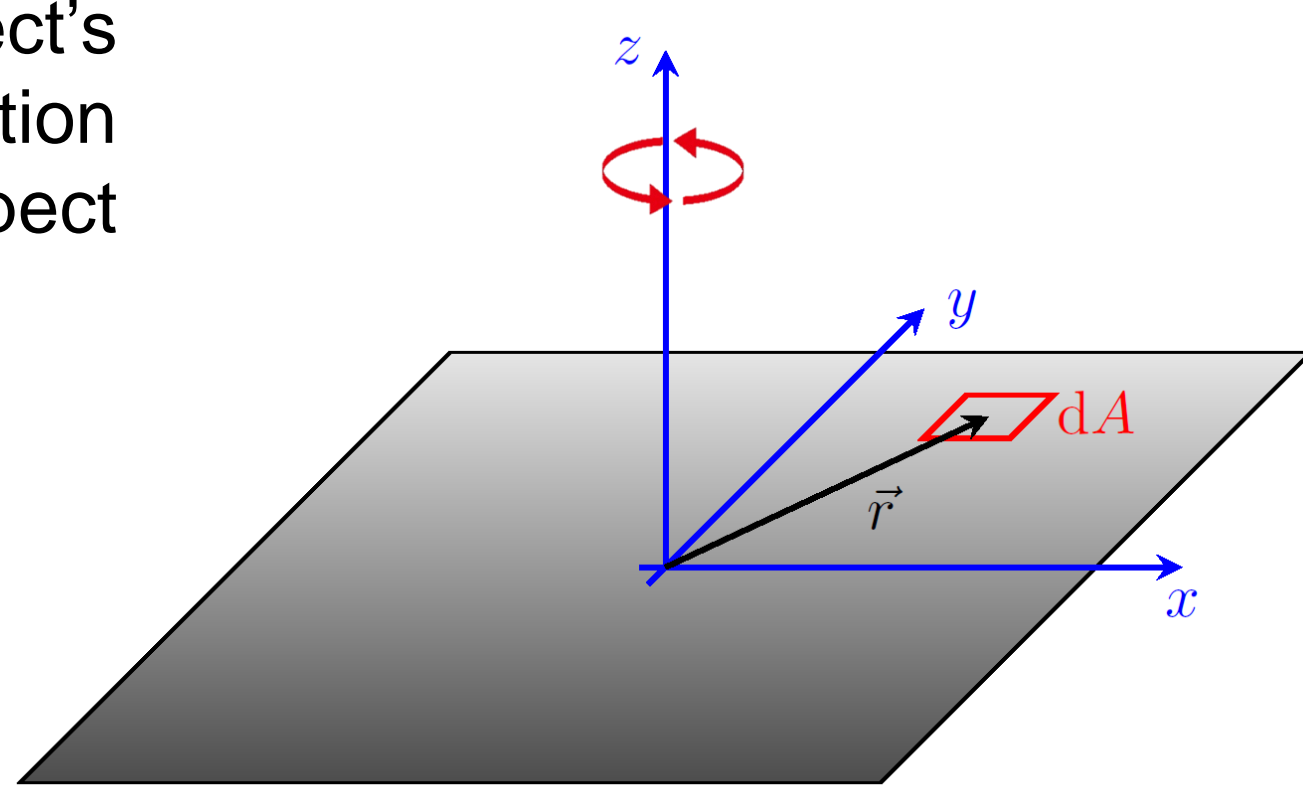


Moment of inertia

The moment of inertia represents an object's resistance to changes in its rotational motion and how its mass is distributed with respect to its axis of rotation.

$$I_Q = \int \vec{r}^2 \rho(\vec{r}) dA$$

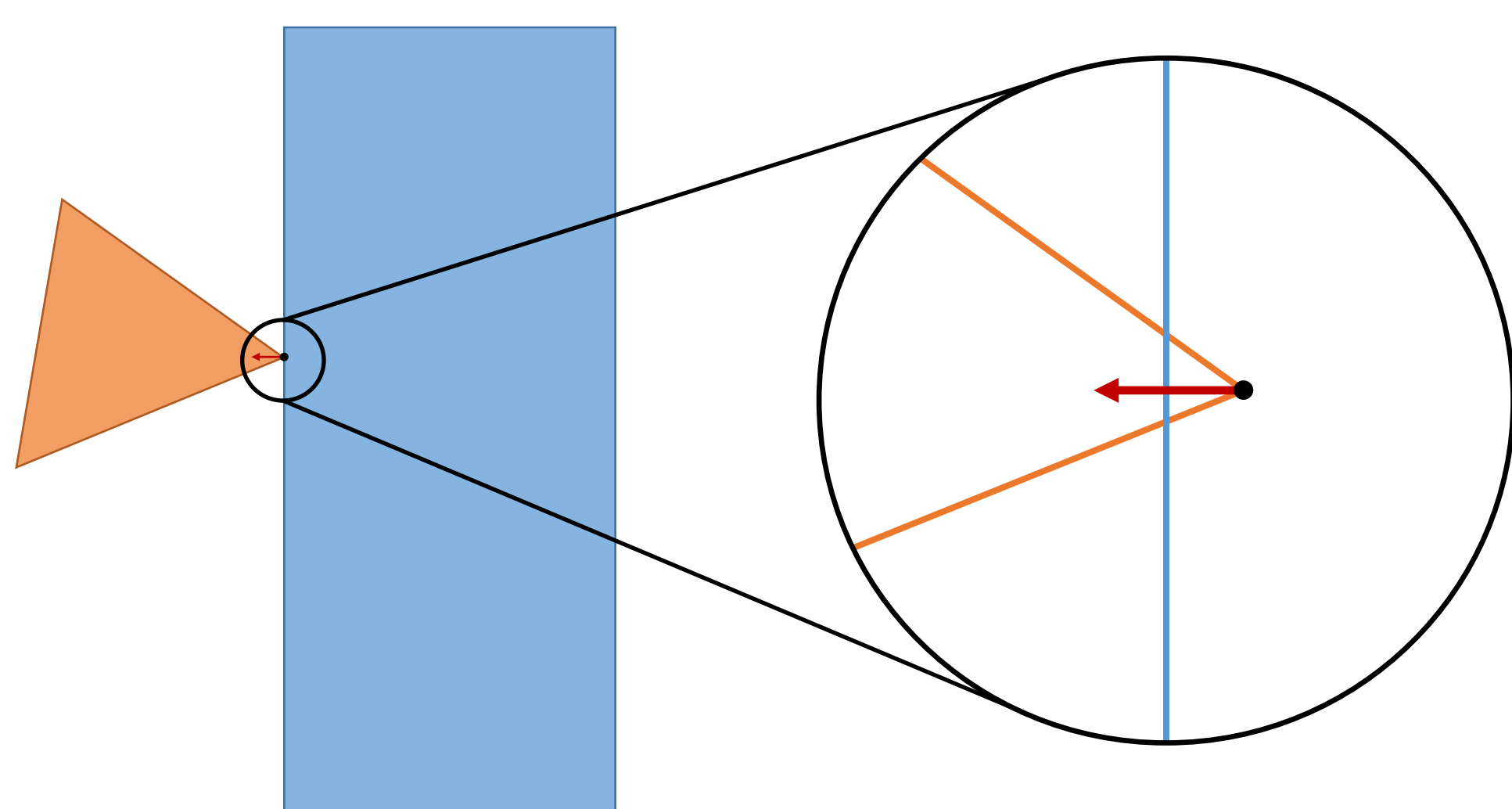
$$= \rho \iint x^2 + y^2 dydx$$



Where $\rho(\vec{r})$ is the density of the polygon Q in position \vec{r} with respect to its barycenter. The general integral (on the first line) is done with respect to the area of the polygon A . In this project, the polygon's density is uniform across its area, thus it can be moved out, and since the engine is in two dimensions, we can simplify \vec{r} to $x^2 + y^2$.

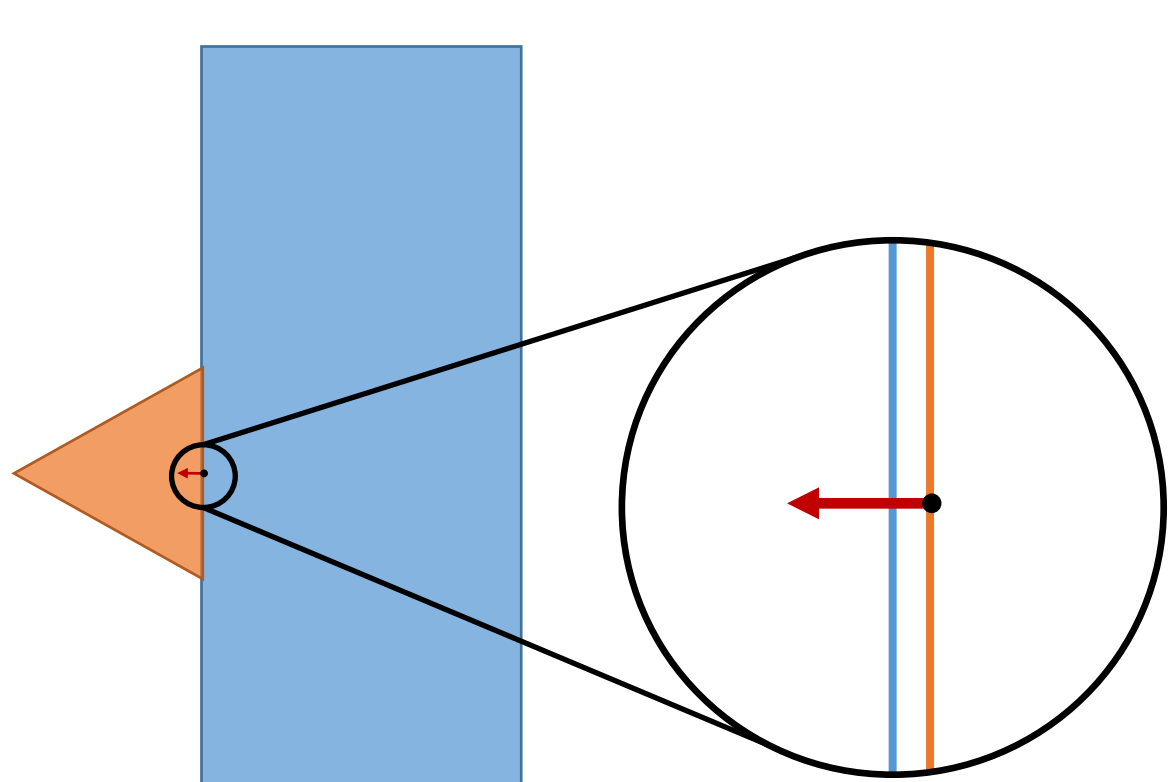
Collision detection

Most common case: vertex on edge

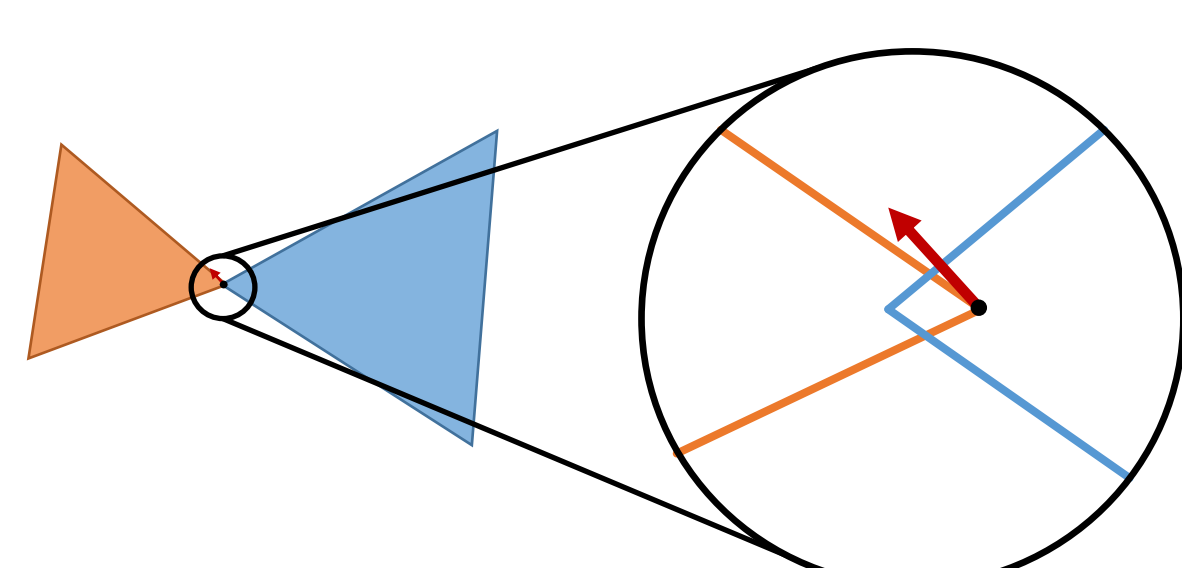


Rare cases: vertex in vertex and parallel

Parallel



Vertex on vertex



Collision Resolution

PHYSICS FORMULAS

$$\vec{v}_{p1} = \vec{v}_{ap1} - \vec{v}_{bp2}$$

$$\vec{v}_{p2} = \vec{v}_{ap2} - \vec{v}_{bp2}$$

$$\vec{v}_{ap1} = \vec{v}_{a1} + \omega_{a1} \times \vec{r}_{ap}$$

$$\vec{v}_{bp1} = \vec{v}_{b1} + \omega_{b1} \times \vec{r}_{bp}$$

$$\vec{v}_{a2} = \vec{v}_{a1} + \frac{j\vec{n}}{m_a}$$

$$\vec{v}_{b2} = \vec{v}_{b1} - \frac{j\vec{n}}{m_b}$$

$$j = \frac{-(1+e) \cdot \vec{v}_{ap1} \cdot \vec{n}}{\frac{1}{m_a} + \frac{1}{m_b} + \frac{(\vec{r}_{ap} \times \vec{n})^2}{I_a} + \frac{(\vec{r}_{bp} \times \vec{n})^2}{I_b}}$$

$$\omega \times \vec{r} = \begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix} \times \begin{pmatrix} r_x \\ r_y \\ 0 \end{pmatrix} = \begin{pmatrix} -\omega r_y \\ \omega r_x \\ 0 \end{pmatrix}$$

$$\vec{v}_{p1} = \vec{v}_{a1} + \omega_{a1} \times \vec{r}_{ap} - \vec{v}_{b1} + \omega_{b1} \times \vec{r}_{bp}$$

$$\vec{v}_{p2} = \vec{v}_{a2} + \omega_{a2} \times \vec{r}_{ap} - \vec{v}_{b2} + \omega_{b2} \times \vec{r}_{bp}$$

$$\omega_{a2} = \omega_{a1} + \frac{\vec{r}_{ap} \times j\vec{n}}{I_a}$$

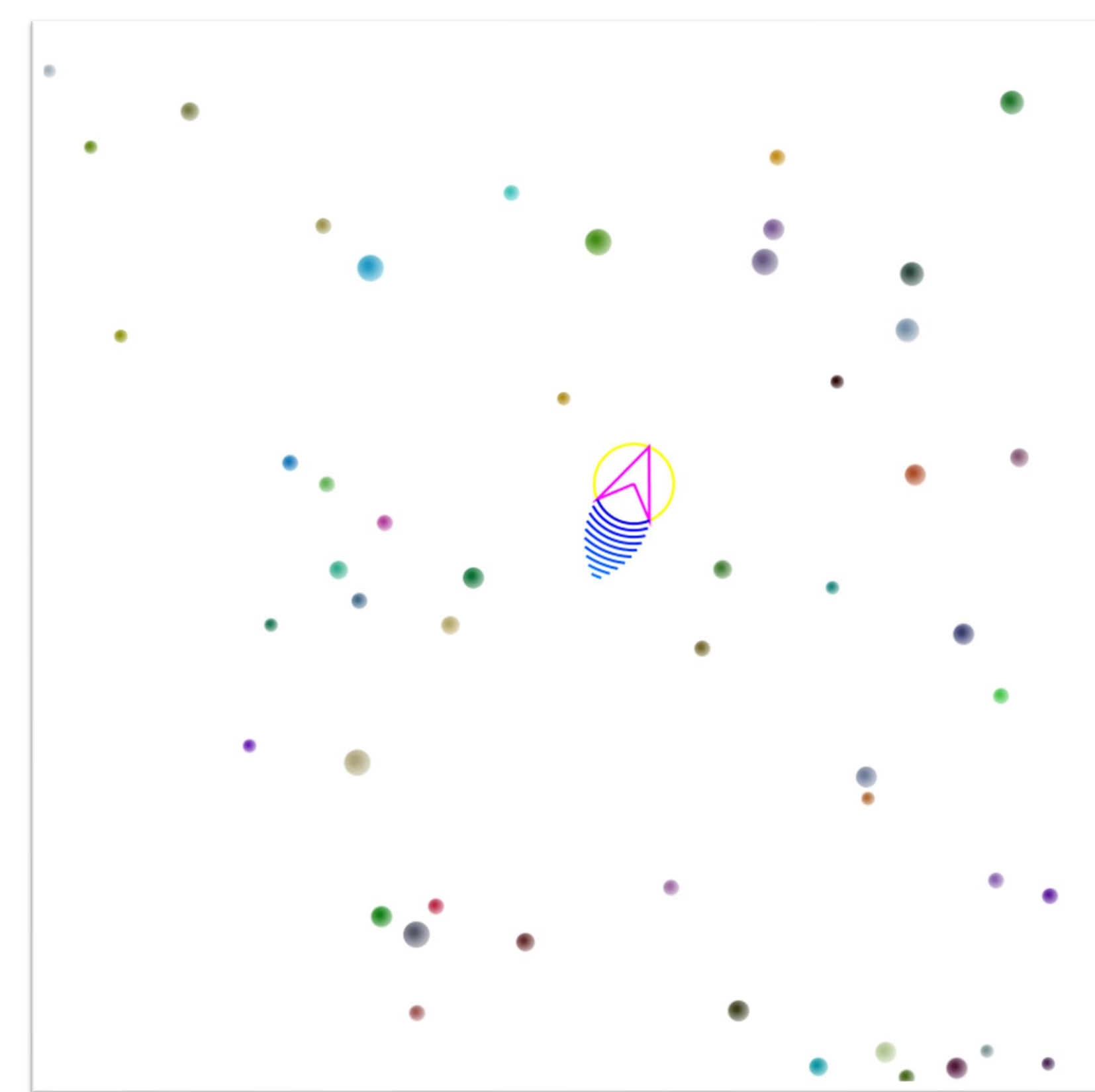
$$\omega_{b2} = \omega_{b1} - \frac{\vec{r}_{bp} \times j\vec{n}}{I_b}$$

$$\vec{v}_{p2} \cdot \vec{n} = -e \vec{v}_{p1} \cdot \vec{n}$$

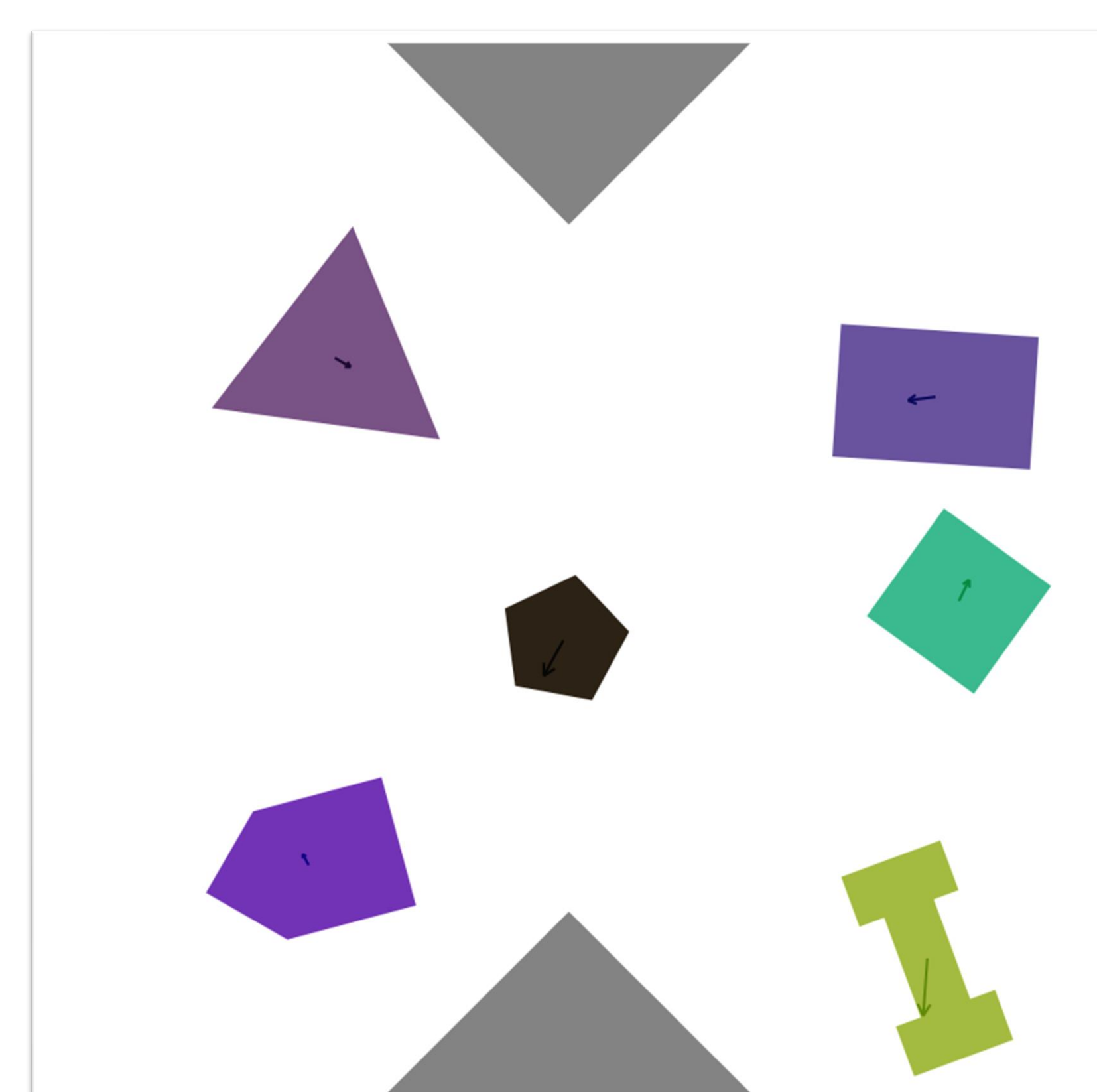
I could write stuff that doesn't make sense, you wouldn't even notice :)

Before vs After Comparison

Flying balls



Colliding polygons



Challenges

- Understanding what the moment of inertia meant
- Finding the formula of the moment for arbitrary polygons
- Handling a collision that spanned over multiple frames:
 - Frame 1: collision detected between A and B, apply resolution
 - Frame 2: collision detected again between A and B, but no resolution must occur, because it already was applied on previous frame, so we need to "ignore" the collision. Finding the "ignoring" criteria was quite difficult.